SIR ARTHUR LEWIS COMMUNITY COLLEGE

ACADEMIC YEAR (2024/2025) – SEMESTER ONE

END OF SEMESTER EXAMINATION

COURSE CODE	:	**MAT242**
COURSE TITLE	:	**Complex Numbers**
LECTURER(S)	:	**Vercilli Gustave-Lord**
DATE	:	**10th December 2024**
TIME	:	**9 a.m. – 11:30 a.m.**
DURATION	:	**2 ½ Hours**
STUDENT ID #	:	_____

GENERAL INFORMATION AND INSTRUCTIONS

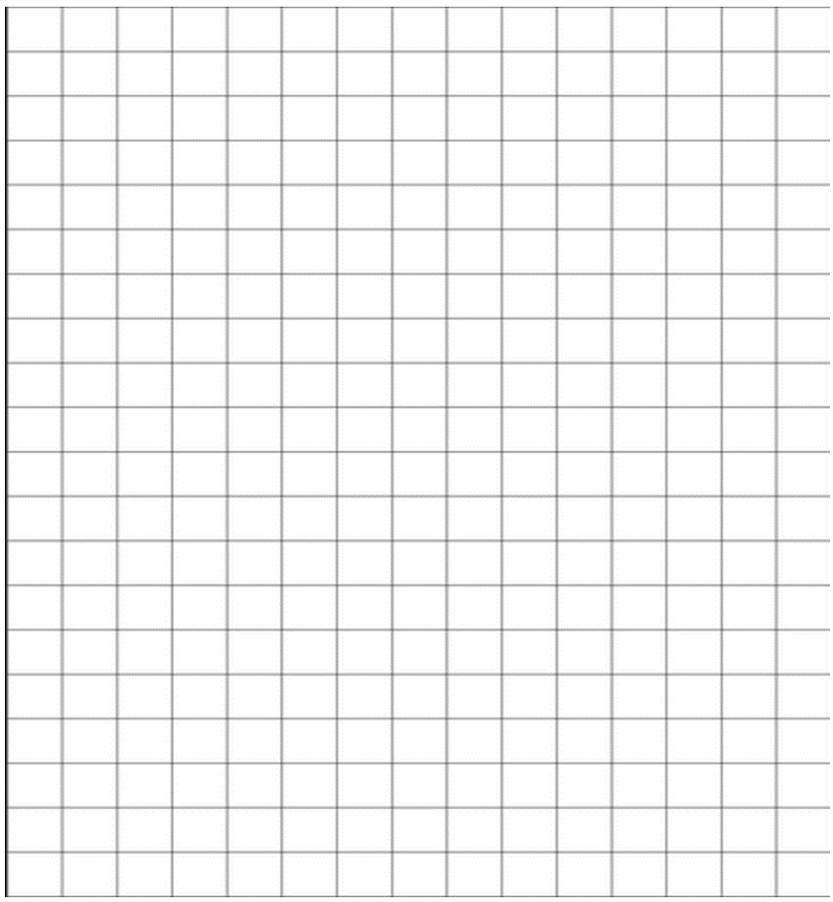
- Students must sign **IN** and **OUT** on the examination class list.
- Write your ID number on the question paper.
- This paper consists of 5 questions. Answer **ALL** questions. Answer each question **IN BLACK OR BLUE PEN** in the space provided.
- Do **not** use correction fluid.

Question #	Maximum Mark	Student Mark
1.	**11**	
2.	**7**	
3.	**9**	
4.	**4**	
5.	**13**	
TOTAL	**44**	

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

1. The loci C_1 and C_2 are given by $|z-3|=3$ and $arg(z-1)=\frac{1}{4}\pi$ respectively.

(a) Sketch, in the space provided, the single Argand diagram of the loci C_1 and C_2.[5]



(b) Find the Cartesian Equation of the locus C_2. [4]

(c) Indicate, by shading, the region of the Argand diagram for which

$$|z-3| \leq 3 \quad \text{and} \quad 0 \leq arg(z-1) \leq \frac{1}{4}\pi.$$

[2]

2. A sequence is given by $u_{n+1} = 2u_n + 3, \quad u_1 = 7, \quad n \geq 1$. Prove by Mathematical Induction that the general formula for the sequence is

$$u_n = 5(2^n) - 3.$$

[7]

3.

(a) Show that $\frac{1}{r} - \frac{1}{r+2} \equiv \frac{2}{r(r+2)}$. [1]

(b) Hence, find an expression, **without simplifying**, in terms of n, for

$$\sum_{r=1}^{n} \frac{2}{r(r+2)}.$$

[6]

(c) Hence, find

$$\sum_{r=1}^{\infty} \frac{2}{r(r+2)}.$$

[2]

4. Find at least the first seven terms of the recurrence relation

$$a_1 = 2,\ a_2 = 1,\ a_n = -a_{n-2} + a_{n-1}, n \geq 3$$

to determine whether the sequence is convergent, divergent or periodic. [4]

5. The complex number z is defined by $z = \frac{a + 2i}{a - i}, a \in \mathbb{R}, a > 0$. Given that the **real** part of z is $\frac{1}{2}$, find

   (a) The value of a, [8]

(b) The argument of z, giving your answer in radians to two decimal places. [5]